

# Homological stability for symmetric diffeomorphism groups and parametrised connected sum

Talk at the GeMAT seminar, IMAR // Martin Palmer-Anghel // 23 August 2018

## Abstract:

One very successful tool for studying the homology of diffeomorphism or mapping class groups of manifolds is *homological stability*: if the diffeomorphism (mapping class) groups of a sequence of manifolds are homologically stable, this reduces the calculation of their homology groups, in a range of degrees, to the calculation of the homology in the limit, which typically has more structure (for example that of a Hopf algebra), and is more amenable to explicit calculations.

There are many results in the literature<sup>1</sup> on the homological stability of diffeomorphism or mapping class groups of sequences of manifolds of the form  $D^{p+q}\sharp(S^p \times S^q)\sharp(S^p \times S^q)\sharp \dots$ , obtained by iterating the operation of connected sum with a product of spheres, for various different values of  $(p, q)$ . A recent result of Tillmann [T] extends this to much more general sequences of manifolds of the form  $W\sharp N\sharp N\sharp \dots$ , at the expense of passing to certain subgroups of the full diffeomorphism groups of these manifolds, namely their *symmetric diffeomorphism groups*.

I will present a generalisation [P.II] of this result, where the operation  $-\sharp-$  of connected sum is generalised to *parametrised connected sum*  $-\sharp_L-$  along a submanifold  $L$ , an operation that includes surgery and Dehn surgery as special cases.

A key input for the proof is homological stability for *moduli spaces of disconnected submanifolds* [P.I], a generalisation of configuration spaces whose points consist of configurations of isotopic copies of a given manifold  $L$  in the ambient manifold. I will give an overview of the main steps of the proof: first, how to reduce [P.II] to [P.I], and then the key ideas of the proof of [P.I].

Note: this is an expanded version of [this](#) talk, given in Salvador, Bahia, Brazil on 26 July 2018.

## References:

- [P.I] M. Palmer. *Homological stability for moduli spaces of disconnected submanifolds, I*, arXiv:[1805.03917](https://arxiv.org/abs/1805.03917), (2018).
- [P.II] M. Palmer. *Homological stability for moduli spaces of disconnected submanifolds, II – Symmetric diffeomorphism groups and parametrised connected sum*, arXiv:[1807.07558](https://arxiv.org/abs/1807.07558), (2018).
- [T] U. Tillmann. *Homology stability for symmetric diffeomorphism and mapping class groups*. *Math. Proc. Cambridge Philos. Soc.* **160**.1 (2016), pp. 121–139. { arXiv:[1510.07564](https://arxiv.org/abs/1510.07564) }

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<sup>1</sup> Including (among others) results of Harer, Ivanov, Boldsen, Hatcher-Wahl, Galatius-Randal-Williams, Perlmutter.