Stability and stable homology for moduli spaces of disconnected submanifolds

Configuration spaces of indistinguishable point-particles on manifolds are interesting spaces from many different viewpoints, including knot theory (via the braid groups), homotopy theory and algebraic geometry. Their homology groups are known to stabilise (if the manifold is connected and open) as the number of point-particles goes to infinity, and their limiting homology is also known. After briefly reviewing these classical results of D. McDuff and G. Segal, I will discuss what is known in the more general setting where one replaces configurations of point-particles with configurations of disjoint unions of closed submanifolds of specified diffeomorphism type and isotopy class. In particular, I will discuss in this setting: (1) homological stability under a certain dimension condition (and corollaries for moduli spaces of manifolds with singularities), and (2) work in progress on identifying the limiting homology.