

On the lower central series of braid groups and their relatives

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Abstract.

Every group has a natural descending filtration that measures its failure to be abelian: its *lower central series*. Understanding this filtration and its associated Lie algebra can give deep information about the underlying structure of the group. How much information it contains depends on the answer to the first basic question that one could ask about this filtration: does it stop? I will talk about recent joint work with Jacques Darné and Arthur Soulié, in which we give a comprehensive answer to this question for *partitioned braid groups* \mathbf{B}_λ for any partition $\lambda \vdash n$ (which interpolate between braid groups and pure braid groups) and their “relatives”, including:

- partitioned *virtual* braid groups \mathbf{vB}_λ ,
- partitioned *welded* braid groups \mathbf{wB}_λ ,
- partitioned *surface* braid groups $\mathbf{B}_\lambda(S)$ for any surface S .

The answer in the last case turns out to depend very subtly on the partition λ of the strands and the topology of the underlying surface S , with the two most difficult cases being the 2-sphere \mathbb{S}^2 and the projective plane \mathbb{RP}^2 . There are three main techniques that we use in our analysis of the lower central series, and I will give an example illustrating the use of each one.

Based on joint work with Jacques Darné and Arthur Soulié — [arXiv:2201.03542](https://arxiv.org/abs/2201.03542)