Scientific report for project 58PCE/2023

Martin Palmer-Anghel

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- Webpage: mdp.ac/pce2023
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- Title: ***The homology and representation theory of moduli spaces***

Executive summary

- **3 publications:**
  - *The pro-nilpotent Lawrence-Krammer-Bigelow representation*
    M. Palmer, A. Soulié
    Journal of Pure and Applied Algebra vol. 229 no. 6 (2025) Article no. 107952
  - *Compact and finite-type support in the homology of big mapping class groups*
    M. Palmer, X. Wu
    Journal of the London Math. Soc. vol. 112 no. 3 (2025) Article no. e70258
  - *Heisenberg homology on surface configurations*
    C. Blanchet, M. Palmer, A. Shaukat
    Mathematische Annalen vol. 393 (2025) pp. 1989–2056
- **5 preprints:**
  - C. Anghel, *Geometric universal Jones invariant from configurations on ovals in the disc*, arXiv:2401.17245 (v2: May 2025)
  - C. Anghel, *Universal geometrical link invariants*, arXiv:2505.18108 (May 2025)
  - A. Măcinic, J. Vallès, *Addition-deletion for conic-line arrangements with split Chern polynomial*, arXiv:2510.02771 (Oct. 2025)
  - M. Palmer, X. Wu, *Embedding groups into acyclic groups*, arXiv:2510.16879 (Oct. 2025)
  - E. Lindell, A. Soulié, *Twisted homological stability for handlebody mapping class groups*, arXiv:2510.21383 (Oct. 2025)
- **3 projects in progress** (in addition to those mentioned above)
- **3 prizes awarded to team members:**
  - *Simion Stoilow* prize of the Romanian Academy: Cristina Palmer-Anghel
  - *Lucian Bădescu* prize, IMAR: Cristina Palmer-Anghel
  - *Lucian Bădescu* prize, IMAR: Martin Palmer-Anghel
- **Project seminar "*Moduli and Friends*":**
  - Webpage: mdp.ac/pce2023/seminar.html
  - **9** talks by external speakers in 2025
- **Conference "*Bucharest topology days*"**
  - Webpage: mdp.ac/seminars/Bucharest-topology-days
  - 21–22 July 2025
- **Activities of team members at conferences, external seminars and scientific visits:**
  - **6** talks at international conferences (**13** participations at international conferences in total)
  - **7** invited talks at seminars
  - **8** scientific visits
  - **3** visits of project collaborators to IMAR

Contents

1. Summary of progress and activities

1.1. Publications

- *The pro-nilpotent Lawrence-Krammer-Bigelow representation*
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- *Compact and finite-type support in the homology of big mapping class groups*
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- *Heisenberg homology on surface configurations*
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1.2. Preprints

- *Geometric universal Jones invariant from configurations on ovals in the disc*
  C. Anghel
  arXiv:2401.17245 (v2: May 2025)
- *Universal geometrical link invariants*
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- *Addition-deletion for conic-line arrangements with split Chern polynomial*
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- *Twisted homological stability for handlebody mapping class groups*
  E. Lindell, A. Soulié
  arXiv:2510.21383 (Oct. 2025)

1.3. Prizes

- *Simion Stoilow* prize of the Romanian Academy: Cristina Palmer-Anghel
- *Lucian Bădescu* prize, IMAR: Cristina Palmer-Anghel
- *Lucian Bădescu* prize, IMAR: Martin Palmer-Anghel

1.4. Project seminar and conference

As part of the grant project, we organised an online seminar called *Moduli and Friends II* (webpage: mdp.ac/pce2023/seminar.html), which is the successor of a previous seminar series called *Moduli and Friends*. During 2025 we had the following invited speakers at the seminar:

- Takuro Abe (Rikkyo University)
- Alexis Aumonier (Stockholm University)

- Louis Hainaut (University of Chicago)
- Anh Trong Nam Hoang (Northeastern University)
- Lukas Kühne (Bielefeld University)
- Paolo Salvatore (University of Rome Tor Vergata)
- Ismael Sierra (University of Toronto)
- Anderson Vera (Universidad Nacional de Colombia)
- Lukas Woike (Université Bourgogne Europe)

In addition, we organised a 2-day conference called *Bucharest Topology Days* at IMAR on 21–22 July 2025, featuring talks by three of the team members, as well as the following external speakers:

- George Altmann (University of Leeds)
- Mihai-Cosmin Pavel (IMAR, Bucharest)
- Awais Shaukat (Namal University, Mianwali)
- Xiaolei Wu (Fudan University, Shanghai)

1.5. Talks at conferences

- Cristina Palmer-Anghel, "A topological model for the HOMFLY-PT polynomial", *Journées de topologie algébrique, géométrique et quantique en Picardie II*, Amiens, 5–7 March 2025.
- Arthur Soulié, "Some stable twisted cohomology computations for handlebody mapping class groups", *Algebraic approaches to mapping class groups of surfaces*, University of Tokyo, 19–23 May 2025.
- Martin Palmer-Anghel, "The homology of Thompson-like groups via algebraic K-theory and étale groupoids", *Journée de topologie quantique*, Dijon, 26 May 2025.
- Cristina Palmer-Anghel, "Universal link invariants via configuration spaces", *Conference on Modern Developments in Low-Dimensional Topology*, ICTP, Trieste, 9–13 June 2025.
- Anca Măcinic, "Curve arrangements. Freeness and geometric constraints", *Moduli, Hilbert Schemes and Singularities*, Cluj-Napoca, 8–12 September 2025.
- Cristina Palmer-Anghel, "Universal link invariants via configuration spaces", *AMS Special Session on Quantum Supergroups in Low-Dimensional Topology*, 25–26 October 2025.

1.6. Participation in conferences (in addition to those mentioned above)

- Martin Palmer-Anghel, *36th YaMCATS meeting*, Leeds, 24 January 2025.
- Martin Palmer-Anghel, *Journées de topologie algébrique, géométrique et quantique en Picardie II*, Amiens, 5–7 March 2025.
- Arthur Soulié, *Journées de rencontre des doctorants du RT2171*, Amiens, 9–11 April 2025.
- Martin Palmer-Anghel, *Conference on Modern Developments in Low-Dimensional Topology*, Trieste, 9–13 June 2025.
- Martin Palmer-Anghel, *Workshop: Blooming Beasts: A conference on the geometry, topology, and dynamics of infinite-type surfaces*, Casa Matemática Oaxaca, 23–27 June 2025.
- Martin Palmer-Anghel, *Rencontre 2025 de Topologie algébrique*, Marseille, 28–31 October 2025.
- Arthur Soulié, *Rencontre 2025 de Topologie algébrique*, Marseille, 28–31 October 2025.

1.7. Invited talks at seminars

- Martin Palmer-Anghel, "Homological stability for asymptotic monopole moduli spaces", *Geometry and Analysis Seminar*, University of Leeds, 19 February 2025.
- Martin Palmer-Anghel, "Vanishing results in the homology of big mapping class groups and Thompson-like groups", *Geometric Group Theory seminar*, University of Cambridge, 21 February 2025.
- Martin Palmer-Anghel, "Nullité et indénombrabilité dans l'homologie des groupes de difféotopie des surfaces de type infini", *Séminaire GAAO*, Clermont-Ferrand, 15 April 2025.
- Arthur Soulié, "Some stable twisted cohomology for handlebody mapping class groups", *Algebra/Topology seminar*, University of Copenhagen, 28 April 2025.

- Cristina Palmer-Anghel, "Invariants universels d'entrelacs via des espaces de configurations", *Séminaire Géométrie, Algèbre, Dynamique et Topologie*, Dijon, 19 June 2025.
- Arthur Soulié, "Surface braid and mapping class group homological representations", *Northeastern Topology Seminar*, Northeastern University, Boston, 16 September 2025.
- Cristina Palmer-Anghel, "Universal link invariants via configuration spaces", *Brussels-London geometry seminar*, University College London, 18 September 2025.

1.8. Scientific visits

- Martin Palmer-Anghel, Isaac Newton Institute, University of Cambridge (research collaboration with Ulrike Tillmann), 20–28 February 2025.
- Arthur Soulié, Université Clermont Auvergne, Clermont-Ferrand (research collaboration with Martin Palmer-Anghel), 23–29 March 2025.
- Arthur Soulié, University of Copenhagen (research collaboration with Erik Lindell), 27 April – 3 May 2025.
- Arthur Soulié, University of Tokyo (research collaboration with Nariya Kawazumi), 13–28 May 2025.
- Martin Palmer-Anghel, Institut de Mathématiques de Bourgogne, Dijon (research collaboration with Lukas Woike), 19–20 June 2025.
- Arthur Soulié, Institut de Mathématique de Marseille, Aix-Marseille Université (research collaboration with Christine Vespa), 13–28 July 2025.
- Arthur Soulié, Northeastern University, Boston (research collaboration with Anh Trong Nam Hoang), 13–20 September 2025.
- Arthur Soulié, University of Chicago (research collaboration with Andreas Stavrou), 20–27 September 2025.

1.9. Visits of project collaborators to IMAR

- Peter Patzt (University of Oklahoma), 27 June – 3 July 2025.
- Awais Shaukat (Namal University, Mianwali), 2 July – 31 August 2025.
- Xiaolei Wu (Fudan University, Shanghai), 19–27 July 2025.

2. Scientific results

2.1. The pro-nilpotent Lawrence-Krammer-Bigelow representation

M. Palmer, A. Soulié

Journal of Pure and Applied Algebra vol. 229 no. 6 (2025) Article no. 107952

We construct a 3-variable enrichment of the Lawrence-Krammer-Bigelow (LKB) representation of the braid groups, which is the limit of a *pro-nilpotent tower of representations* having the original LKB representation as its bottom layer. More precisely, we prove:

Theorem *There is a pro-nilpotent tower of representations of the classical braid groups* $\mathbf{B}_n$ *whose* $\ell = 2$ *layer is the Lawrence-Krammer-Bigelow representation.*

The levels of pro-nilpotent towers of representations are indexed by integers $l \geqslant 2$. The limit as $l \to \infty$ of this tower is a representation of $\mathbf{B}_n$ defined over the integral group ring $\mathbb{Z}[\mathbf{RB}_2]$, where $\mathbf{RB}_2$ is the two-strand ribbon braid group. This is a special case of a more general construction:

Theorem *For each* $k \geqslant 2$*, there is a representation of* $\mathbf{B}_n$ *defined over the integral group ring* $\mathbb{Z}[\mathbf{RB}_k]$ *that recovers the* k*th Lawrence-Bigelow representation, defined over* $\mathbb{Z}[\mathbb{Z}^2]$*, when reduced along the abelianisation* $\mathbf{RB}_k \twoheadrightarrow (\mathbf{RB}_k)^{ab} = \mathbb{Z}^2$*.*

In the case $k = 2$, we compute explicit matrices, over the group ring $\mathbb{Z}[\mathbf{RB}_2]$, describing the action of the standard generators σ_i of $\mathbf{B}_n$. Since $\mathbf{RB}_2 = \mathbb{Z}^2 \rtimes \mathbb{Z}$, these give a non-commutative 3-variable enrichment of the classical Lawrence-Krammer-Bigelow representation.

In addition, we also construct analogous pro-nilpotent towers of representations of *surface braid groups* and *loop braid groups*.

2.2. Compact and finite-type support in the homology of big mapping class groups

M. Palmer, X. Wu

Journal of the London Math. Soc. vol. 112 no. 3 (2025) Article no. e70258

For any infinite-type surface S, a natural question is whether the homology of its mapping class group $\mathrm{Map}(S)$ contains any non-trivial classes that are supported on either (i) a *compact* subsurface or (ii) a *finite-type* subsurface. We give an almost-complete answer to these questions when the genus of S is positive (including infinite) and a partial answer when the genus of S is zero. Our methods involve the notion of shiftable subsurfaces as well as homological stability for mapping class groups of finite-type surfaces.

More precisely, let us say that a homology class $\alpha \in H_*(\mathrm{Map}(S))$ has *compact support* (resp. *finite-type support*) if there is a compact (resp. finite-type and properly embedded) subsurface $\Sigma \subset S$ such that α is in the image of $H_*(\mathrm{Map}_\partial(\Sigma)) \to H_*(\mathrm{Map}(S))$, where $\mathrm{Map}_\partial(-)$ means mapping classes fixing boundary pointwise and $\mathrm{Map}_\partial(\Sigma) \to \mathrm{Map}(S)$ is defined by extending homeomorphisms by the identity.

For an infinite-type surface S, the questions mentioned above may be stated precisely as follows:

Question Are there non-zero homology classes in $H_*(\mathrm{Map}(S))$ with compact (resp. finite-type) support?

If we write $\mathrm{Map}_{\mathfrak{C}}(S)$ (resp. $\mathrm{Map}_{\mathfrak{F}}(S)$) for the colimit of $\mathrm{Map}_{\partial}(\Sigma)$ over all compact (resp. finite-type and properly embedded) subsurfaces $\Sigma \subset S$, then there are natural maps

$$\mathrm{Map}_{\mathfrak{C}}(S) \longrightarrow \mathrm{Map}_{\mathfrak{F}}(S) \longrightarrow \mathrm{Map}(S)$$

and the question is equivalent to asking whether these maps induce non-trivial maps on homology. When S has infinite genus, we have the following result for compactly supported classes:

Theorem *Let S be an infinite-genus surface. Then the map induced on homology by $\mathrm{Map}_{\mathfrak{C}}(S) \to \mathrm{Map}(S)$ is zero, when taking coefficients in any field.*

The answer for finite-type support is a little more complicated. A *puncture* of S is an end that is both planar and isolated (i.e. not a limit point of other ends). The number of punctures of S is always finite or countably infinite. A *mixed end* of S is one that is both non-planar and a limit point of punctures. (Note that the existence of a mixed end implies that the number of punctures is infinite.)

Theorem *Let S be an infinite-genus surface with $p \in \{0, 1, 2, \ldots, \infty\}$ punctures. Then the map induced on homology by $\mathrm{Map}_{\mathfrak{F}}(S) \to \mathrm{Map}(S)$ is:*

(if $p = 0$)	*zero, when taking coefficients in any field.*
(if $0 < p < \infty$)	*non-zero, when taking coefficients in any abelian group.*
(if $p = \infty$ and $\exists$ mixed end)	*zero, when taking coefficients in any field.*

If S is an infinite-type surface of *finite but non-zero* genus, then we prove that $\mathrm{Map}(S)$ always has non-zero homology classes with compact (and therefore finite-type) support.

If S is an infinite-type surface of *genus zero*, then the answer is much more complicated, and depends subtly on the topology of the space of ends of S. For example, if the number of punctures of S is finite but at least 4, then $\mathrm{Map}(S)$ has non-zero homology classes with compact support. In the other direction, we prove for $S = \mathbb{C} \smallsetminus \mathbb{Z}$ that $\mathrm{Map}(S)$ does not have any non-zero homology classes with compact support (when taking coefficients in any field).

2.3. Heisenberg homology on surface configurations

C. Blanchet, M. Palmer, A. Shaukat
Mathematische Annalen vol. 393 (2025) pp. 1989–2056

Motivated by the *Lawrence-Krammer-Bigelow representations* of the classical braid groups, we study the homology of unordered configurations in an orientable genus-g surface with one boundary component, over non-commutative local systems defined from representations of the *discrete Heisenberg group*.

Mapping classes act on the local systems and for a general representation of the Heisenberg group we obtain a representation of the mapping class group that is twisted by this action. For the linearisation of the affine translation action of the Heisenberg group we obtain a genuine, untwisted representation of the mapping class group. In the case of the generic *Schrödinger representation*, by composing with a Stone-von Neumann isomorphism we obtain a projective representation by bounded operators on a Hilbert space, which lifts to a representation of the stably universal central extension of the mapping class group. We also discuss the finite dimensional Schrödinger representations, especially in the even case. Based on a natural intersection pairing, we show that our representations preserve a sesquilinear form.

We moreover provide an upper bound on the kernels of our twisted representations of mapping class groups: they are contained in the intersection of the *Magnus kernel* and the nth term of the *Johnson filtration*.

Finally, in the simplest non-trivial case of our construction (genus 1 and configurations of 2 points), we also explicitly compute matrices for the twisted representation over the non-commutative ground ring $\mathbb{Z}[u^{\pm 1}]\langle a^{\pm 1}, b^{\pm 1}\rangle/(ab = u^2 ba)$. For example, the matrix describing the action of the boundary Dehn twist is 3×3 and its entries are polynomials in the non-commutative variables a, b, u involving approximately 180 terms in total (when fully simplified), exhibiting the richness of these representations already in this simple case.

2.4. Geometric universal Jones invariant from configurations on ovals in the disc

C. Anghel
arXiv:2401.17245 (v2: May 2025)

We construct geometrically a *universal Jones invariant* as a limit of invariants given by graded intersections in configuration spaces. For any fixed level $\mathcal{N}$, we define a new knot invariant, called the $\mathcal{N}^{th}$ *unified Jones invariant*, globalising topologically all coloured Jones polynomials at levels less than $\mathcal{N}$. It is defined via the intersection points between Lagrangian submanifolds supported on arcs and ovals in the disc. The geometry of these Lagrangians is novel: previous topological models involved immersed submanifolds rather than embedded ones. We do this by defining a new local system that refines the Lawrence representation, and depends of the distribution of multiplicities of points in the configuration space on the ovals.

On the algebraic side, Habiro's famous invariant for knots is a universal invariant globalising the family of coloured Jones polynomials. He conjectured that this universal invariant recovers also the ADO invariant divided by the Alexander polynomials, which was proved by Willetts in a version of Habiro's ring. The universal Jones invariant that we construct belongs to a different ring that comes with a map to Habiro's ring.

We prove that our invariant recovers this version of Habiro's invariant. The difference is that our invariant is given as a limit of new knot invariants, the $\mathcal{N}^{th}$ unified Jones invariants. These invariants in turn provide a geometrical understanding of sets of all coloured Jones polynomials of bounded colour, collecting more information as we increase the colour.

2.5. Universal geometrical link invariants

C. Anghel
arXiv:2505.18108 (May 2025)

We construct geometrically two universal link invariants: a *universal ADO invariant* and a *universal Jones invariant*, as limits of invariants given by graded intersections in configuration spaces. More specifically, for a fixed level $\mathcal{N}$, we define new link invariants: the $\mathcal{N}^{th}$ *unified Jones invariant* and the $\mathcal{N}^{th}$ *unified Alexander invariant*. They globalise topologically all coloured Jones polynomials for links with multi-colours bounded by $\mathcal{N}$ and all ADO polynomials with bounded colours. These invariants both come from the same weighted Lagrangian intersection supported on configurations on arcs and ovals in the disc.

The question of providing a universal non semi-simple link invariant that recovers all of the ADO polynomials was an open problem. A parallel question about semi-simple invariants for the case of knots is the subject of Habiro's famous universal knot invariant. Habiro's universal construction is well-defined for knots and can be extended just for certain classes of links. Our universal Jones link invariant is defined for any link and recovers all coloured Jones polynomials, providing a new semi-simple universal link invariant. The first non semi-simple universal link invariant that we construct unifies all ADO invariants for links, answering the open problem about the globalisation of these invariants.

2.6. Addition-deletion for conic-line arrangements with split Chern polynomial

A. Măcinic, J. Vallès
arXiv:2510.02771 (Oct. 2025)

We present combinatorial/geometric obstructions induced by the factorisation over the integers of the Chern polynomial of the bundle of logarithmic vector fields associated to a complex projective plane curve. Our results generalise at the same time similar results on projective line arrangements whose characteristic polynomial factors over the integers and results on free curves. We give a splitting criterion for a rank 2 vector bundle, in terms of restrictions to smooth conics.

2.7. Embedding groups into acyclic groups

M. Palmer, X. Wu
arXiv:2510.16879 (Oct. 2025)

We show that labelled Thompson groups and twisted Brin–Thompson groups are all acyclic. This allows us to prove several new embedding results for groups:

- First, every group of type F_n embeds quasi-isometrically as a subgroup of an acyclic group of type F_n that has no proper finite-index subgroups. This improves results of Baumslag–Dyer–Heller ($n = 1$) and Baumslag–Dyer–Miller ($n = 2$) from the early 80s, as well as a more recent result of Bridson ($n = 2$).
- Second, we show that every finitely generated group embeds quasi-isometrically as a subgroup of a 2-generated, simple, acyclic group.

Our results also allow us to produce, for each $n \geqslant 2$, the first known example of an acyclic group that is of type F_n but not F_{n+1}. These examples can moreover be taken to be simple. Furthermore, our examples provide a rich source of universally boundedly acyclic groups.

2.8. Twisted homological stability for handlebody mapping class groups

E. Lindell, A. Soulié
arXiv:2510.21383 (Oct. 2025)

We prove twisted homological stability for *handlebody mapping class groups*. Using the categorical framework developed by Randal-Williams and Wahl, we establish that the homology of the handlebody mapping class groups stabilises with respect to both genus and the number of marked boundary discs, for all coefficient systems of finite degree. Our first main theorem refines and extends the twisted stability result for handlebodies outlined by Randal-Williams and Wahl, allowing any number of marked discs and boundary points. We then introduce the notion of *coefficient bisystem* to treat stability under variation of boundary markings. As an application, we deduce homological stability for moduli spaces of 3-dimensional handlebodies equipped with tangential structures.

2.9. Heisenberg homological representations for non-orientable mapping class groups

M. Palmer, A. Shaukat
Project in progress

We are working on adapting §2.3 to the setting of mapping class groups of *non-orientable surfaces*. We have constructed a non-orientable analogue of the Heisenberg group as a quotient of (pure) braid groups on non-orientable surfaces. The kernel of this quotient is preserved by the mapping class group action, so we obtain twisted homological representations of the non-orientable mapping class groups, analogous to the orientable setting. We are studying methods for untwisting these twisted representations, as well as investigating upper bounds on their kernels.

2.10. Stable homology of configuration-section spaces

M. Palmer, U. Tillmann

Project in progress

In this project we study *configuration-section spaces*: these are moduli spaces parametrising configurations of particles in an underlying manifold together with a "field" (a section of a bundle) defined on the complement of the configuration. When the underlying manifold is the complex plane $\mathbb{C}$ these spaces recover the classical Hurwitz spaces. In previous joint work we proved that configuration-section spaces exhibit homological stability whenever the underlying manifold is connected and has non-empty boundary, subject to a strong condition on the charges of the particles (in the 2-dimensional setting this corresponds to a condition on the monodromy of ramified coverings). Continuing in this direction, we are working on calculating the *stable homology* of configuration-section spaces. An argument for this was sketched in a withdrawn preprint of Ellenberg, Venkatesh and Westerland in the more restrictive setting of *configuration-mapping spaces*; however, there is a significant technical issue with their approach and we are working instead on a different strategy, inspired by the approach of Galatius-Madsen-Tillmann-Weiss for cobordism categories, to identify the stable homology in our setting.

2.11. On the homology of generalised Röver-Nekrashevych groups

M. Palmer, X. Wu

Project in progress

As a complement to our recent joint work §2.7, which used methods of topological groupoids and a recent theorem of Xin Li, we are working on directly adapting the K-theoretic methods of Szymik and Wahl (in their groundbreaking proof of acyclicity of Thompson's group V) to calculate the homology of *generalised Röver–Nekrashevych groups*, including *labelled Higman–Thompson groups*.

Project director,

C.S. I Dr. Martin Palmer-Anghel